MULTIMEDIA   UNIVERSITY

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

TMA1101 – Calculus
(All sections / Groups)

12 MARCH 2019
2.30pm – 4.30pm
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **5** pages with **5** questions only excluding the cover page.

2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided.

4. **No calculators are allowed.**

QUESTION 1 [10 MARKS]

(a) Find the following limits.
   [***You must show at least one intermediate step where*** $\lim\limits_{x \to c}$ ***is still needed.***]

   (i) $\lim\limits_{x \to 5} \dfrac{x^2 - 25}{2x - 10}$

   (ii) $\lim\limits_{x \to \infty} \dfrac{3x^2 + \cos x}{2 + x^2}$

                                                                              [2 marks]

(b) Given $f(x) = \begin{cases} 3 + x & x < 4 \\ 2x & x = 4 \\ x^2 - 9 & x > 4 \end{cases}$

   (i)   Find $f(4)$.

   (ii) Determine $\lim\limits_{x \to 4^-} f(x)$ and $\lim\limits_{x \to 4^+} f(x)$.

         [***For this part, you must show at least one intermediate step where*** $\lim\limits_{x \to 4^-}$ ***or*** $\lim\limits_{x \to 4^+}$
         is still needed.]

   (iii) Does $\lim\limits_{x \to 4} f(x)$ exist? Give your reason. If it exists, state its value.

   (iv) Is the function $f(x)$ continuous at $x = 4$? Give your reason for your answer.

                                                                              [4.5 marks]

(c) (i)   State the intermediate value theorem (i.e., the full statement including the hypothesis and the conclusion).

   (ii) Show that there is a root of the equation $2x^3 + 5x^2 - x - 7 = 0$ in the interval [1, 2]. You must write proper steps to arrive at conclusion; just writing some calculations would not be enough.

                                                                              [3.5 marks]

Continued......

QUESTION 2 [10 MARKS]

(a) Use the formal definition of the derivative to compute $f'(-2)$ when $f(x) = x(x+2)$.
   You are reminded to write proper steps.

                                                                              [2.5 marks]

(b) Find $\dfrac{dy}{dx}$ with y as given.

   [Use the product rule or the quotient rule for differentiation; show proper steps.]

   (i) $y = e^{3x}\left(1 + \sqrt{x}\right)$

   (ii) $y = \dfrac{2 + \sin x}{x^2}$

                                                                              [3 marks]

(c) The point $(2, -1)$ lies on the curve $3y^2 - 2xy + 5x = 17$.

   Use implicit differentiation to obtain $\dfrac{dy}{dx}$ **in terms of x and y.**

   Then determine the gradient of the tangent to the curve $3y^2 - 2xy + 5x = 17$ at the point $(2, -1)$.

                                                                              [4.5 marks]

Continued......

QUESTION 3 [10 MARKS]

(a) (i)   Use $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$ and $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$ to find the values of A and B which make the equation $\cos4\theta\cos2\theta = A\cos6\theta + B\cos2\theta$ an identity.

     (ii)   Evaluate $\displaystyle\int_{0}^{\frac{\pi}{6}} \cos4x\cos2x\, dx$

[3.5 marks]

(b) (i)   Determine the values of A and B in the following partial fraction decomposition.
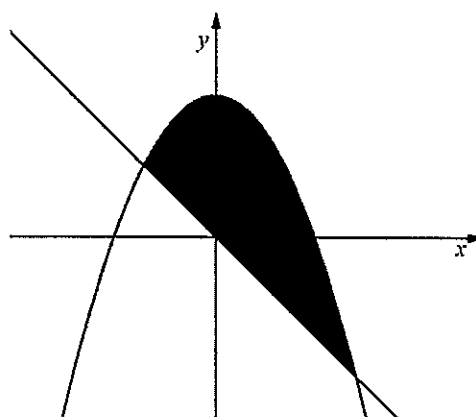$$\frac{5x-8}{x^2-2x-8} = \frac{A}{x+2} + \frac{B}{x-4}$$

     (ii)   Integrate
$$\int \frac{5x-8}{x^2-2x-8}\,dx$$

[3 marks]

(c) The figure shows a region bounded by the parabola $y = 2 - x^2$ and the straight line $y = -x$.

     (i)   Determine the x-coordinates of the points of intersection between the parabola and the straight line.

     (ii)   Write down a definite integral that can be used to find the area of this region and proceed to find the area.

[3.5 marks]

Continued......

QUESTION 4 [10 MARKS]

(a) Given the sequence $\{a_n\}$ with $a_n = \dfrac{1+2n^2}{3n^2+5}$.

Determine $\lim\limits_{n\to\infty} a_n$; you are reminded to write proper steps.

Then determine whether the infinite series $\sum\limits_{n=1}^{\infty} \dfrac{1+2n^2}{3n^2+5}$ is convergent.

Give the reason for your answer.

[2 marks]

(b) Use the ratio test to determine whether the infinite series $\sum\limits_{n=1}^{\infty} \dfrac{4^n}{n^3}$ is convergent.

[3 marks]

(c) Find the **Maclaurin polynomial** of order 3 for $f(x) = \dfrac{1}{(1+x)^3}$.

[3 marks]

(d) A periodic function $f(x)$ with period 2π is defined as

$$f(x) = \begin{cases} 0 & -\pi \leq x < 0 \\ 2 & 0 \leq x < \pi \end{cases}$$

The Fourier series of $f(x)$ has the form $f(x) = \dfrac{a_0}{2} + \sum\limits_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$.

Determine the value of a_3.

[2 marks]

Continued......

QUESTION 5 [10 MARKS]

(a) Given $F(x,y) = 2y^2 + \sin x - e^x \ln y$, find the partial derivatives $\dfrac{\partial F}{\partial x}$ and $\dfrac{\partial F}{\partial y}$.

[1 mark]

(b) Solve the first order separable equation $\dfrac{dy}{dx} = \dfrac{3x^2 - 1}{\cos y}$ subject to the initial condition $y(2) = 0$. You may leave your answer in implicit form.

[2.5 marks]

(c) You are told that e^{3x} is an integrating factor for the first order linear equation $\dfrac{dy}{dx} + 3y = \dfrac{7}{e^{3x}}$ subject to the initial condition $y(0) = 1$.
Solve the equation and give your solution in explicit form.

[3 marks]

(d) (i) Find the roots of the characteristics equation of the homogeneous differential equation $y'' - 5y' - 14y = 0$. Then write down the complementary function y_h of this homogeneous equation.

(ii) Find the particular equation of the second order differential equation $y'' - 5y' - 14y = e^{-2x}$.

(iii) Hence, write down the general solution for the differential equation $y'' - 5y' - 14y = e^{-2x}$

[3.5 marks]

End of Page.